Mark Scheme (Results)

Summer 2023

Pearson Edexcel International GCSE In Mathematics A (4MA1) Paper 2H

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC – special case
    - oe – or equivalent (and appropriate)
    - dep – dependent

- indep – independent
- awrt – answer which rounds to
- eeoo – each error or omission

- **No working**

  If no working is shown, then correct answers normally score full marks.

  If no working is shown, then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams) and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question. E.g. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review. If there is a choice of methods shown, mark the method that leads to the answer on the answer line; where no answer is given on the answer line, award the lowest mark from the methods shown.

  If there is no answer on the answer line, then check the working for an obvious answer.

- **Parts of question**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another,

Brackets and speech marks:

0.32 × 200 (= 64) the brackets here mean that the calculation is required for the mark and not the answer – however the answer would also secure the mark. If a student gave 0.32 × 200 = 68 they would still gain the mark as the method is correct and does not require the calculation to be correct for the award of the mark.

64 alone would also gain the mark.

200 – "146"
This shows that the calculation requires 200 minus the calculation that gave 146; if the calculation was shown but inaccurately worked out then the method mark would still be gained.

Eg 146 should have come from 0.73 × 200
If the student had given 0.73 × 200 = 156 and then given 200 – 156 this would have gained the method mark…. the 156 came from a correct calculation even though the arithmetic was incorrect.

International GCSE Maths				
Apart from questions 1, 15c, 21, 23, 25 the correct answer, unless clearly obtained by an incorrect method, should be taken to imply a correct method				
Q	**Working**	**Answer**	**Mark**	**Notes**
1	eg $\frac{14}{3}$ **and** $\frac{6}{5}$		3	M1 both fractions expressed as correct improper fractions, no need for ÷ or × may be equivalent to those given eg $\frac{70}{15}$ or $\frac{18}{15}$ etc. A student could invert $\frac{6}{5}$ and go straight to the 2nd M1, this mark is then implied.
	$\frac{14}{3}\times\frac{5}{6}$ oe **or** $\frac{70}{15}\div\frac{18}{15}$			M1 For inverting 2nd fraction and showing intention to multiply **or** for both fractions expressed as correct equivalent fractions with the same denominator with intention to divide eg $\frac{70}{15}\div\frac{18}{15}$
	eg $\frac{14}{3}\times\frac{5}{6}=\frac{70}{18}=\frac{35}{9}=3\frac{8}{9}$ **or** $\frac{14}{3}\times\frac{5}{6}=\frac{70}{18}=3\frac{16}{18}=3\frac{8}{9}$ **or** $\frac{\cancel{14}^{7}}{3}\times\frac{5}{\cancel{6}^{3}}=\frac{35}{9}=3\frac{8}{9}$ **or** $\frac{14}{3}\div\frac{6}{5}=\frac{70}{15}\div\frac{18}{15}=\frac{70}{18}=\frac{35}{9}=3\frac{8}{9}$ **or** correct working to $\frac{35}{9}$ **and** writing $3\frac{8}{9}=\frac{35}{9}$ (may be earlier in working) *working required*	Shown		A1 Dep on M2 for conclusion to $3\frac{8}{9}$ from correct working – either sight of the result of the multiplication or division e.g. $\frac{70}{18}$ must be seen or correct cancelling prior to the multiplication to $\frac{35}{9}$ **OR** writing $3\frac{8}{9}=\frac{35}{9}$ (maybe on first line of working) and correct working as far as LHS $=\frac{35}{9}$ **NB: marks are awarded for use of fractions not decimals (but allow a decimal check of answer)**
				Total 3 marks

2	$1-(0.32+0.13+0.28)$ oe eg 1 – 0.73 (= 0.27) or 0.32 × 200 (= 64) or 0.13 × 200 (= 26) or 0.28 × 200 (= 56) or 0.73 × 200 (= 146)		3	M1	(0.27 may be seen in table) [could work with percentages eg 100 – 32 – 13 – 28 (=27)]
	[1 – "0.73"] × 200 oe eg "0.27" × 200 or 200 – "64" – "26" – "56" or 200 – "146"			M1	for a complete method or for an answer of $\frac{54}{200}$
	Correct answer scores full marks (unless from obvious incorrect working)	54		A1	
					Total 3 marks

3	$(4x - 27) + (3x + 46) = 180$ oe or "expression for C" + $(3x + 10) = 180$ **or** $7x + 19 = 180$ **or** $3x + 46 + 4x - 27 + 3x + 10 + [\text{"}180 - (3x + 10)\text{"}] = 360$		4	M1	Sum angles *A* and *B* to 180, or find an expression for *BCD* and sum all angles to 360. [condone missing brackets and condone use of any letter or expression for angle *C* (even *x* or *BCD*)]
				A1	$x = 23$
	eg $3 \times$"23" + 46 (= 115) **or** eg $180 - (3 \times$"23" + 10) (= 101)			M1ft	dep on M1 using **their *x*** to calculate a value for angle *B* or 'their' *C* (cannot be a negative value and cannot just be *x*)
	Correct answer scores full marks (unless from obvious incorrect working)	115		A1	Allow $3x + 46$ or *ABC* if 115 is clearly seen in working or on diagram
					Total 4 marks

4 (a)	x	−3	−2	−1	0	1	2	3	2	B2 for all correct values, otherwise B1 for 3 or 4 correct values
	y	**8**	2	**−2**	**−4**	−4	**−2**	**2**		
(b)									2	M1 dep on B1 scored in (a) for at least 5 points plotted correctly (ft their table)
					correct curve					A1 for a fully correct curve (all coordinates correct and correctly plotted and joined with a curve and curved between (0, –4) and (1, –4))
										Total 4 marks

5	2 and 15 seen or 1×2 (+) 3×5 (= 17)	$2x + 15x$ (= 85) or $\frac{2}{3}y + 5y(=85)$ or $0.25t \times 2 + 0.75t \times 5$ (= 85)		4	M1 For 2 and 15 oe seen or 17 or a correct equation in one unknown for number of 2p coins (x) or number of 5p coins (y) or total number of coins (t)
	$85 \div (2 + 15)$ (= 5) or at least two pairs of multiples of the values of 2 and 15 (eg 4, 30; 6, 45…..) or 10(p) (and) 75(p) or 10 : 75 or 5×2 and 15×5 $2 \times 5 + 5 \times 3 \times 5$ or 20 coins	$17x = 85$ ($x = 5$) or $\frac{17}{3}y = 85$ ($y = 15$) or $4.25t = 85$ ($t = 20$)			M1 Assumes previous M1 for number of 2p coins **or** number of 5p coins **or** total number of coins **or** value of 2p coins **and** value of 5p coins may be **clearly** listed eg 2 555 2 555 2 555 2 555 2 555 with no ambiguity
	5 (2p coins) and 15 (5p coins) or 5 : 15 (if clearly identified (or used) as the key ratio eg not just part of a list) or $(3 - 1) \times 5$	eg $15 - 5$ oe			M1 Correct number of 2p coins **and** 5p coins or a sum to find the difference in number of coins
	Correct answer scores full marks (unless from obvious incorrect working)		10		A1 SCB1 if no other marks awarded for 21.25 in working or on answer line
					Total 4 marks

6 (a)		7.6×10^7	1	B1	
(b)		0.000 54	1	B1	
					Total 2 marks

7	*DCO* = 90 (or right (angle)) or *DAO* = 90 (or right (angle)) Could also be awarded for *CAO* + *CAD* = 90 or *DAC* + *CAO* = 90		3	M1	may be marked on diagram – also allow right angle 'square' symbol on diagram	M2 implied by 360 – 90 – 90 – 48 **or** 360 – 228
	Obtuse *AOC* = 360 – 90 – 90 – 48 (= 132)oe **or** Obtuse *AOC* = 2(180 – (0.5 × 48) – 90) (= 132) **or** Obtuse *AOC* = 180 – "24" – "24" or 180 – 48 (if working with $\triangle DAC$ and $\triangle OAC$) **or** Reflex *AOC* = 90 + 90 + 48			M1	**dep on M1** being awarded may be marked on diagram	
	Correct answer scores full marks (unless from obvious incorrect working)	228		A1	SC if no other marks awarded 132 gains B1	
						Total 3 marks

8	for 0.04 × 680 oe **(** = 27.2) **or** 1.04 × 680 oe **(** = 707.2)		3	M1	For finding 4% or 104% of the value	**or** M2 for 680 × 1.04^3 **or** 680× 1.04^4 **or** 795.50…..
	1.04 × "707.2" (= 735.488) oe **and** 1.04 × "735.488" (= 764.90752) oe or 0.04 ×(680 + "27.2") = 0.04 ×"707.2" = 28.288 0.04 ×"(707.2 +28.288)"= 0.04 ×"735.488" = 29.41952 "735.488" + "29.41952" = 764.90752….			M1	for completing the method	
	Correct answer scores full marks (unless from obvious incorrect working)	765		A1	or 764 – 765 (if a correct answer is seen in working and then rounded incorrectly, award full marks) **SC:** if no other marks gained award M1 for 1.12 × 680 oe **or** 761.6(0) (or 762) **or** 0.12 × 680 oe or 81.6 (or 82) **or** 0.96^3 × 680 oe or 601.62… (or 602) (accept (1 + 0.04) as equivalent to 1.04 throughout but not (1 + 4%))	
						Total 3 marks

9	For 27×1000 ($= 27\,000$) **or** $\frac{27}{60\times 60}$ ($= 0.0075$ or $\frac{3}{400}$) **or** $\frac{1000}{60\times 60}$ ($\frac{5}{18} = 0.27(7...)$) **or** sight of 450		3	M1	For one of $\times 1000$ (eg sight of 27 000) or ($\div 60 \div 60$) or $\div 3600$ oe ie correct conversion of distance units or of time units or $\frac{1000}{60\times 60}$	M2 for $27 \div 3.6$ **or** $27 \times \frac{5}{18}$
	$\frac{27\times 1000}{60\times 60}$ oe eg $(0.45 \times 1000) \div 60$ **or** $0.27\ldots. \times 27$			M1	For a fully correct method with correct use of brackets eg $27\,000 \div 60 \times 60$ is M1 only if not recovered	
	Correct answer scores full marks (unless from obvious incorrect working)	7.5		A1	oe eg $\frac{15}{2}$ or $7\frac{1}{2}$ oe	
						Total 3 marks

10	17×11 (= 187) **or** 18.5×12 (=222) **or** 18×9 (=162) **or** 18.5×10 (= 185)		4	M1	Expression for total of **A** or **B** either including or excluding last round	M2 for $1.5 \times 11 + 18.5$ (= 35) **or** $9 \times 0.5 + 18.5$ (= 23) OR 1.5×11 (= 16.5) **or** 0.5×9 (= 4.5)
	$18.5 \times 12 - 17 \times 11$ ("222" – "187")(= 35) **or** $18.5 \times 10 - 18 \times 9$ ("185" – "162")(= 23) **or** $\frac{"187"+x}{12} = 18.5$ ($x = 35$) **or** $\frac{"162"+y}{10} = 18.5$ ($y = 23$) **or** Diff between **A** and **B** in first rounds "187" – "162" (= 25) **or** Diff between **A** and **B** after further round "222" – "185" (= 37) [or 2×18.5 (= 37) (2 must come from correct working)]			M1	expression for number of points gained by **A** or **B** in the last round or for an equation that could lead to the number of points gained by **A** or **B** in the last round	
	"35" – "23" or "37" – "25" or "16.5" – "4.5"			M1	calculation for difference between number of points scored in last round	
	Correct answer scores full marks (unless from obvious incorrect working)	12		A1		
						Total 4 marks

The 2 is 2 further rounds of 18.5 ie 37 comes from $18.5 \times 12 - 18.5 \times 10$ so the 2×18.5 is $(12 - 10) \times 18.5$

11	eg $DEK = \frac{360}{9}$ **or** 40 **or** interior angle = $\frac{(9 - 2) \times 180}{9}$ **or** 140 or $OFK = 140 \div 2$ (= 70) or $FOK = \frac{2}{9} \times 360$ (= 80) or $EDK = 180 - 0.5 \times 140$ (=110) Angles marked correctly (any exterior or interior angle) gains this mark		3	M1	method to find interior or exterior angle or correct interior or exterior angle stated or shown correctly on diagram or for using 70° for *OFK* or 80° for *FOK* or 110 for *EDK* If a student has only found an interior or exterior angle and has clearly mixed up interior and exterior angles this mark cannot be awarded but can still award for any of the others angles stated
	$EDK = 110$ and $DEK = 40$ or $FOK = 80$ and $OFK = 70$ or $ODE = 70$ and $DEK = 40$ or $FED = 140$ and $EDK = 110$ oe			M1	For two correct angles that can lead directly to the answer in a single step (eg 180 – both angles or one angle minus the other)
	Correct answer scores full marks (unless from obvious incorrect working)	30		A1	
					Total 3 marks

12	$\cos BAD = \frac{8}{14}$ **or** $\sin ABD = \frac{8}{14}$ **or** $\sin ABD = \frac{8\sin 90}{14}$ **or** $(BD =) \sqrt{14^2 - 8^2} \left(= \sqrt{132} = 2\sqrt{33} = 11.4(89...)\right)$		4	M1
	$BAD = \cos^{-1}\left(\frac{8}{14}\right)$ $(= 55.(15\ldots))$**or** $\cos^{-1}\left(\frac{14^2 + 8^2 - "11.489"^2}{2\times 14\times 8}\right)$ $BAD = \sin^{-1}\left(\frac{"11.489..."}{14}\right)$ $(= 55.(15\ldots))$ **or** $BAD = \tan^{-1}\left(\frac{"11.489..."}{8}\right)(= 55.(15\ldots))$ **or** $BAD = 180 - 90 - \sin^{-1}\left(\frac{8}{14}\right)(= 180 - 90 - 34.8... = 55.(15...)$ oe **or** $CAD = 180 - 38 - \sin^{-1}\left(\frac{8}{14}\right) - 90$ $(= 180 - 38 - 34.8 - 90 = 17.2)$			M1
	$\tan("55.15..." - 38) = \frac{CD}{8}$ oe eg $\tan 17.2 = \frac{CD}{8}$ $\frac{CD}{\sin(55.1... - 38)} = \frac{8}{\sin(90 - (55.1... - 38)}$ oe			M1 A correct equation with *CD* being the only unknown value
	Correct answer scores full marks (unless from obvious incorrect working)	2.47		A1 2.44 – 2.48
				Total 4 marks

13 (a)		$\frac{4}{6}$ $\frac{1}{6},\frac{5}{6},\frac{1}{6},\frac{5}{6}$	2	B2oe	B1 for $\frac{4}{6}$ (or $\frac{2}{3}$) on LH bottom branch B1 for all other branches correct (allow 0.66 or 0.67 or better, 0.16 or 0.17 or better, 0.83 or better)
(b)	$"\frac{4}{6}"\times"\frac{5}{6}"$		2	M1ft	ft their tree diagram if probabilities less than 1 (only considering this product or 1 – (RR + RY + YR))
	Correct answer scores full marks (unless from obvious incorrect working)	$\frac{5}{9}$		A1	oe eg $\frac{20}{36}$ or (0.55(55…) or 55% or better or 56%
					Total 4 marks

14	(a) (i)		90	2	B1	
	(a) (ii)		Angle in a semicircle is 90° oe Angle in a semicircle is 90° oe Triangle in semicircle is 90° oe Angle at centre is double (oe eg ×2) angle at circumference oe Angle at circumference is half (oe) angle at centre. oe		B1	dep on B1 in (a)(i) Valid reason given, underlined words give minimally acceptable answer.
	(b) (i)		22	2	B1	
	(b) (ii)		Angles in the same segment (are equal) **or** angles at the circumference subtended from the same arc of the circle **or** angles on the same chord		B1	dep on B1 in (b)(i) Valid reason given, underlined words give minimally acceptable answer.
						Total 4 marks

15	(a)		$8a^9$	2	B2	for a fully correct answer. if not B2, then B1 for 8 or a^9 as part of a product in answer, or final line of working
	(b)		$1000x^3$	2	B2	for a fully correct answer. (B1 for final answer or final line of working with: 1000 or x^3 as part of a product or $(10x)^3$ or $\frac{1}{1000x^3}$)
	(c)	eg $30\times\frac{1-2y}{3} = 30\times\frac{4}{5} - 30\times\frac{2y-1}{2}$ oe or eg $\frac{10(1-2y)}{30} = \frac{6\times4}{30} - \frac{15(2y-1)}{30}$ oe or eg $\frac{1-2y}{3} = \frac{2\times4}{10} - \frac{5(2y-1)}{10}$ oe or eg $10(1-2y) = 3\times2\times4 - 3\times5(2y-1)$ oe or eg $\frac{10(1-2y)+15(2y-1)}{30} = \frac{4}{5}$ or $\frac{2(1-2y)}{6} + \frac{3(2y-1)}{6} = \frac{4}{5}$ oe (as above)		3	M1	For clear intention to multiply all terms by 30 (or 3 × 5 × 2) or a multiple of 30 oe in an equation or to express all terms over 30 (or 3 × 5 × 2) or a multiple of 30 oe in an equation **or** writing RHS over 10 or a multiple of 10 or 'cross multiplying' in an equation **or** bringing terms in y on LHS side and leaving $\frac{4}{5}$ on RHS and writing terms on LHS over 6 or a multiple of 6 in an equation [if expanded numerators, allow one error]
		eg $10 - 20y = 24 - 30y + 15$ oe eg $10y = 29$ or $50 - 100y + 150y - 75 = 120$ oe or $10 - 20y + 30y - 15 = 24$ oe $2 - 4y + 6y - 3 = 4.8$			M1	(ft if only one error) Expanding brackets and multiplying by denominator with no more than one error in total
		Working required	2.9		A1	oe eg $\frac{29}{10}$ or $2\frac{9}{10}$ dep on M2
						Total 7 marks

16 (a)	$Q = k\sqrt{t}$		3	M1	for linking Q and t correctly (must have constant eg k) (allow $Q \propto k\sqrt{t}$)
	eg $6 = k\sqrt{4}$ or $3 = k\sqrt{1}$ or $k = 3$			M1	for substituting a suitable pair of values **or** finding $k = 3$ (allow $\propto$ sign)
	Correct answer scores full marks (unless from obvious incorrect working)	$Q = 3\sqrt{t}$		A1	oe allow q for Q (must have =) allow 2.95 – 3.05 for k if method clearly shown and readings correct ±0.5 small square allow an answer of $Q = k\sqrt{t}$ with $k = 3$ clearly stated
(b)	1.2^2 or 1.44 or 144 could be within a calculation eg $\left(\frac{3\times1.2}{3}\right)^2 (=1.44)$ or $\left(\frac{6\times1.2}{3}\right)^2 (=5.76)$ eg $\left(\frac{6\times1.2}{"3"}\right)^2 \div 4\ (=1.44)$ or $\left(\frac{6\times1.2}{3}\right)^2 - 4 (=1.76)$ eg 6 ×1.2 = 7.2 and reading from $Q = 7.2$ to t axis and calculates t value ÷ 4 (or 0.04)		2	M1ft	ft for an equation in the correct form Stating the multiplier 1.2^2 or 1.44 or 144 or showing the multiplier within a calculation or for using a pair of values and showing a correct calculation for the increase in t, ft their k from (a) or correct calculation and use of graph
	Correct answer scores full marks (unless from obvious incorrect working)	44		A1	allow 43 – 45 as long as not from incorrect working
					Total 5 marks

17 (a)	eg $3x^2-2x+18x-12$ $(=3x^2+16x-12)$ **or** $x^2+6x+6x+36$ $(=x^2+12x+36)$ (allow in a table with no sign indicating +)		3	M1	for a correct method to expand two brackets with at least 3 terms correct out of 4 terms seen (or 2 terms correct out of 3 terms seen)
	eg $3x^3+36x^2+108x-2x^2-24x-72$ or $3x^3+18x^2+18x^2+108x-2x^2-12x-12x-72$ **or** $3x^3+16x^2-12x+18x^2+96x-72$ or $3x^3-2x^2+18x^2-12x+18x^2-12x+108x-72$			M1ft	ft dep on M1 and a quadratic for a correct method to multiply by the 3rd bracket allow one further error
	Correct answer scores full marks (unless from obvious incorrect working)	$3x^3+34x^2+84x-72$		A1	If no working shown then award B2 for 3 out of a maximum of 4 terms correct
	ALTERNATIVE				
	$3x^3-2x^2+18x^2-12x+18x^2-12x+108x-72$		3	M2	For a complete expansion with 8 terms present of which 4 are correct (M1 for 4 correct terms from any number of terms)
		$3x^3+34x^2+84x-72$		A1	
	See next page for 17(b)				

17 (b)	$w^2 = \dfrac{e+g}{ef-d}$		4	M1	for removing square root
	$w^2ef - w^2d = e+g$ oe			M1	for multiplying by denominator and expanding in a correct equation
	$w^2ef - e = g + w^2d$ oe			M1ft	ft their equation dep on 2 terms in e and two other terms for gathering terms in e on one side and other terms the other side
	Correct answer scores full marks (unless from obvious incorrect working)	$e = \dfrac{g+w^2d}{w^2f-1}$		A1	oe eg $e = \dfrac{-g-w^2d}{1-w^2f}$, $e = -\dfrac{g+w^2d}{1-w^2f}$ oe must see $e =$ on answer line or in working.
					Total 7 marks

18		**C**	3	B1 check diagrams
		F		B1 check diagrams
		A		B1 check diagrams
				Total 3 marks

19	$3\left(x^2-2x\right).....$ or $3\left(x^2-2x+.....\right)$ oe or		3	M1	(where …… is any number or no number)
	$3(x-1)^2$…… or $3\left[\left(x-1\right)^2.....\right]$ oe			M1	(where …… is any number or no number)
	Correct answer scores full marks (unless from obvious incorrect working)	$3\left(x-1\right)^2+2$		A1	(if student continues to solve a quadratic equation, ISW)
					Total 3 marks
Alternative mark scheme for 19					
19	$ax^2 -2abx + ab^2 + c$		3	M1	for multiplying out $a(x-b)^2+$ c to obtain $ax^2 -2abx + ab^2 + c$ oe
	Any 2 of the following: $a = 3$ or $2ab = 6$ or $ab^2 + c = 5$ oe			M1	for equating coefficients with any 2 of $a = 3$ or $2ab = 6$ or $ab^2 + c = 5$ oe seen
	Correct answer scores full marks (unless from obvious incorrect working)	$3\left(x-1\right)^2+2$		A1	(if student continues to solve a quadratic equation, ISW)
					Total 3 marks

20	(RRR:) $\frac{3}{12}\times\frac{2}{11}\times\frac{1}{10}$ $(=\frac{1}{220})$ **or** $(0.25 \times 0.18\ldots\times 0.1 = 0.0045\ldots)$ (2R, 1G:) $\frac{3}{12}\times\frac{2}{11}\times\frac{9}{10}$ $(=\frac{9}{220})$ **or** $(0.25 \times 0.18\ldots\times 0.9 = 0.0409\ldots)$ (2G, 1R:) $\frac{3}{12}\times\frac{9}{11}\times\frac{8}{10}$ $(=\frac{36}{220}=\frac{9}{55})$ **or** $(0.25 \times 0.81\ldots\times 0.8 = 0.163\ldots)$ (GGG:) $\frac{9}{12}\times\frac{8}{11}\times\frac{7}{10}$ $(=\frac{84}{220}=\frac{21}{55})$ $(0.25 \times 0.72\ldots\times 0.7 = 0.381\ldots)$		3	M1oe	For an expression to find one of the stated probabilities
	$1-"\frac{1}{220}"$ (1 – "0.0045….") or $"\frac{84}{220}"+3\times"\frac{36}{220}"+3\times"\frac{9}{220}"$ $(0.381\ldots+ 3 \times 0.163\ldots+ 3 \times 0.0409\ldots)$			M1oe	Dep M1 Complete method
	Correct answer scores full marks (unless from obvious incorrect working)	$\frac{219}{220}$		A1oe	0.9954….allow 0.99 (99%) or 0.995 (99.5%)
					Total 3 marks

21	$2(3y-1)^2+3y^2=11$	$2x^2+3\left(\frac{x+1}{3}\right)^2=11$		5	M1	substitution of linear equation into quadratic
	$21y^2-12y-9=0$ oe	$7x^2+2x-32=0$ oe			M1	dep on previous M1 for multiplying out and collecting terms, forming a three term quadratic in any form of $ax^2 + bx + c$ (= 0) with at least 2 coefficients (a or b or c) correct
	eg $(7y+3)(y-1)=0$ $\frac{-(-12)\pm\sqrt{(-12)^2-4\times21\times(-9)}}{2\times21}$ $21\left[(y-\frac{2}{7})^2-\frac{4}{49}\right]-9=0$ oe (gives $y=1, y=-\frac{3}{7}$)	eg $(7x+16)(x-2)=0$ $\frac{-(2)\pm\sqrt{(2)^2-4\times7\times-32}}{2\times7}$ $7\left[(x-\frac{1}{7})^2-\frac{1}{49}\right]-32=0$ (gives $x=2, x=-\frac{16}{7}$)			M1	dep on M1 for solving **their** 3 term quadratic equation using any correct method (if factorising, allow brackets which expanded give 2 out of 3 terms correct) (if using formula allow one sign error in subst terms and some simplification – allow as far as eg $\frac{12\pm\sqrt{144+756}}{42}$ or $\frac{-2\pm\sqrt{4+896}}{14}$)(if completing the square allow as far as shown (allow error in final constant) **or** correct values for x **or** correct values for y
	eg $3\times1-1$ **and** $3\times-\frac{3}{7}-1$	eg $\frac{2+1}{3}$ and $\frac{-\frac{16}{7}+1}{3}$			M1ft	dep on previous M1 for substituting (must be shown) **their** 2 found values of x or y in a suitable equation (use 2dp or better for substitution) **or** fully correct values for the other variable (correct labels for x / y)
	Working required		$x=2, y=1$ **and** $x=-\frac{16}{7}, y=-\frac{3}{7}$		A1	dep on M2 (allow coordinates) must be paired correctly allow $x = -2.28(57\ldots)$ and $y = -0.42(85\ldots)$ (even if obtained from premature rounding of the other variable.)
						Total 5 marks

22	$9^2 = 11^2 + 16^2 - 2 \times 11 \times 16 \times \cos BCA$ oe or $11^2 = 9^2 + 16^2 - 2 \times 9 \times 16 \times \cos BAC$ or $16^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos ABC$ or (area of $\triangle ABC =)\sqrt{18 \times 2 \times 7 \times 9} (= 47.6235...)$ oe		5	M1	For a start to the correct method to find angle *BCA* or angle *BAC* or angle *ABC* or a fully correct method to find the area of the triangle
	$(\cos BCA =)\left(\frac{11^2 + 16^2 - 9^2}{2 \times 11 \times 16}\right) (BCA = 32.763...)$ or $(\cos BAC =)\left(\frac{9^2 + 16^2 - 11^2}{2 \times 9 \times 16}\right) (BAC = 41.409...)$ or $(\cos ABC =)\left(\frac{9^2 + 11^2 - 16^2}{2 \times 9 \times 11}\right) (ABC = 105.826...)$ or $\frac{1}{2} \times 16 \times BD = "47.6235..."$			M1	For a correct rearrangement for cos*BCA* or cos*BAC* or cos*ABC* or a correct equation to find *BD* (accept angles to the nearest whole number rounded or truncated as long as not from incorrect working)
	(*BD* =) 11 sin"32.763..."(= 5.95...) oe eg 11 sin(180 – "41.4…" – 105.8…") (= 5.95…) or 9 sin"41.4…." (= 5.95…) oe or $\frac{"47.6235..." \times 2}{16} (= 5.95...)$ oe or $\sqrt{11^2 - "9.25"^2}$ or $\sqrt{9^2 - "6.75"^2}$ $11 \sin\left(\sin^{-1}\left(\frac{9 \sin "105.826..."}{16}\right)\right)$ (= 5.95…) oe			M1	For a correct calculation that will lead to the value of *BD* "47.6235…" may also come from 0.5 × 9 × 11 × sin"105.8…" or 0.5 × 9 × 16 × sin"41.4…" or 0.5 × 16 × 11 × sin"32.7…" [Students may find an angle by sine rule after already finding an angle and use this]
	$\tan FDB = \frac{10}{"5.95..."}$ oe			M1	For a correct expression for the required angle (in form tan*x* =… or cos*x* =… or sin*x* =…) oe
	Correct answer scores full marks (unless from obvious incorrect working)	59.2		A1	awrt 59.2
	SEE OVER FOR ALTERNATIVE SCHEME				**Total 5 marks**

Angle $DBC = 57.237\ldots$ Angle $ABD = 48.591\ldots$ $AD = 6.75$ m $CD = 9.25$ m

22	$BD^2 = 11^2 - (16-y)^2$ and $BD^2 = 9^2 - y^2$ oe		5	M1	For 2 different expressions in the same single variable for BD or BD^2
	$11^2 - (16-y)^2 = 9^2 - y^2$ ($y = 6.75$ or $x = 9.25$)			M1	Equating the 2 expressions
	$BD = \sqrt{9^2 - (16-"9.25")^2}$ or $\sqrt{11^2 - "9.25"^2}$ $(= 5.95)$			M1	A correct calculation to find BD ("9.25" or "6.75" must come from a correct method)
	$\tan FDB = \frac{10}{"5.95..."}$ oe			M1	For a correct expression for the required angle (in form tan$x$ =… or cos$x$ =… of sin$x$ =…) oe
	Correct answer scores full marks (unless from obvious incorrect working)	59.2		A1	awrt 59.2
					Total 5 marks

23	$(x^2 =)\ \dfrac{13+6\sqrt{5}}{2\sqrt{5}-3}$		4	M1	expression for x^2
	$\dfrac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\dfrac{2\sqrt{5}+3}{2\sqrt{5}+3}$ or $\dfrac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\dfrac{-2\sqrt{5}-3}{-2\sqrt{5}-3}$			M1	dep on previous M1 showing a **correct** product to rationalise the denominator (must be correct x^2)
	eg $\dfrac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\dfrac{2\sqrt{5}+3}{2\sqrt{5}+3}=\dfrac{99+44\sqrt{5}}{11}$ or eg $\dfrac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\dfrac{2\sqrt{5}+3}{2\sqrt{5}+3}=\dfrac{26\sqrt{5}+39+60+18\sqrt{5}}{20-9}$ or eg $\dfrac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\dfrac{2\sqrt{5}+3}{2\sqrt{5}+3}=\dfrac{26\sqrt{5}+39+12(\sqrt{5})^2+18\sqrt{5}}{(2\sqrt{5})^2-3^2}$			M1	dep on previous M1 continuing the expansion of the product on the numerator and denominator – maybe one of these forms or a combination of forms
	Working required	$2+\sqrt{5}$		A1	dep on M3 accept $a = 2$, $b = 5$
					Total 4 marks

24	$\frac{-5-10}{4--2}\ (=-\frac{5}{2})$		6	M1	A correct method to find the gradient of AC
	$y-10=-\frac{5}{2}(x+2)$ oe eg $y=-\frac{5}{2}x+5$ or $y--5=-\frac{5}{2}(x-4)$ oe or $5x+2y=10$ oe			M1	ft (if M1 scored) correct equation of AC
	$y-4=\frac{2}{5}(x--\frac{27}{5})$ oe or $4=\frac{2}{5}\left(-\frac{27}{5}\right)+c\ \left(y=\frac{2}{5}x+6.16\right)$ $\frac{4-y}{-\frac{27}{5}-x}=\frac{2}{5}$ oe or $5y-2x=\frac{154}{5}$ oe			M1	ft (if first M1 scored) equation of BD or correct equation using gradient of BD
	solves $-\frac{5}{2}x+5=\frac{2}{5}x+6.16$ oe eg $\begin{matrix}10x+4y=20\\-10x+25y=154\end{matrix}$ oe, with operation of addition **or** $\begin{matrix}25x+10y=50\\-4x+10y=61.6\end{matrix}$ oe, with operation of subtraction **or** $x=\frac{5}{2}y-\frac{154}{10}$ oe or $y=\frac{2}{5}x+\frac{154}{25}$ oe substituted in other equation			M1	Solve equation OR Solve simultaneously the correct equations of lines of AC and BD or correct equation from gradient or other correct equation. If elimination: same coefficient of x or y with suitable sign used to eliminate. If substitution: x or y substituted into other equation.
	Coordinates of intersection of AC and BD : $x=-\frac{2}{5}, y=6$			M1	oe value of x and y at intersection of AC and BD
	Correct answer scores full marks (unless from obvious incorrect working)	(4.6, 8)		A1	oe coordinates of D
	See next page for working with *AD = AB, CD = CB* or gradients			**Total 6 marks**	

24	eg $(10-4)^2+\left(-2+\frac{27}{5}\right)^2$ $(=47.56)$ $(AB=6.896\ldots)$ **or** eg $(-5-4)^2+(4+\frac{27}{5})^2(=169.36)$ $(CB=13.013\ldots)$ **or** eg $\frac{-5-10}{4--2}$ **or** $\frac{4-y}{-\frac{27}{5}-x}$ oe		6	M1 A correct method to find AB^2 or CB^2 or AB or CB or a correct gradient expression for AC or DB
	eg $(y-10)^2+(x+2)^2=(10-4)^2+\left(-2+\frac{27}{5}\right)^2$ **or** eg $(y+5)^2+(x-4)^2=(-5-4)^2+(4+\frac{27}{5})^2$ **or** $\frac{-5-10}{4--2}\times\frac{4-y}{-\frac{27}{5}-x}=-1$ oe eg $-60+15y=6x+32.4$			M1 Using D (x, y) form a correct equation $AD^2=AB^2$ **or** $CD^2=CB^2$ **or** gradients $AC\times DB=-1$ (Using D (x, y))
	eg $2x-5y=-30.8$ **or** $x=2.5y-15.4$ **or** $y=0.4x+6.16$ oe			M1 uses rearrangement or solving simultaneous equations to find a correct 3 term linear equation
	eg $(y-10)^2+(2.5y-15.4+2)^2=(10-4)^2+\left(-2+\frac{27}{5}\right)^2$ eg $(0.4x+6.16+5)^2+(x-4)^2=(-5-4)^2+(4+\frac{27}{5})^2$			M1 uses substitution to obtain a correct quadratic equation in one unknown
	$7.25y^2-87y+232=0$ oe or $1.16x^2+0.928x-28.8144=0$ oe			M1 for a 3 term quadratic that can be used to find the value of x or the value of y at D
		(4.6, 8)		A1 oe coordinates of D
				Total 6 marks

25	255 or 265 or 2.85 or 2.75		4	B1	for sight of a correct upper or lower bound
	$(V =)\ \frac{4}{3}\pi\times(2.75)^3$ $(=\frac{1331}{48}\pi$ or 87.1137….)			M1	calculation to find V using $V=\frac{4}{3}\pi r_{LB}{}^3$ where $2.75\le r_{LB}<2.8$ or use of 2.85
	$(D =)\ \frac{265\pi}{\frac{4}{3}\times\pi\times2.75^3}$ (condone missing π for 265π (also may have cancelled out π))			M1	method to find UB of density, using LB of V and UB of M for correct substitution into $D=\frac{\pi M_{UB}}{V_{LB}}$ where $260<M_{UB}\le265$ **and** $87.11...\le V_{LB}<91.95...$ oe
		9.56		A1	dep on M2 and all correct bounds used allow 9.55 - 9.56
					Total 4 marks

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